1. Is it possible that an event is independent of itself? If so, when?

Let A be an event. If A is independent of itself, then P(A) = P(A\A) = P(A)2, so P(A) is 0 or 1. So this is only possible in the extreme cases that the event has probability 0 or 1.

1. Is it always true that if A and B are independent events, then Ac and Bc are independent events? Show that it is, or give a counterexample.

Let A and B be independent events, and let A and C be independent events. How do I show that A and B∪C are independent events as well?

According to the definition of independent events, A and B∪C are independent if and only if

P(A∩(B∪C))=P(A)P(B∪C).

Since A and B and A and C are independent, I know that

P(A∩B)=P(A)P(B) and P(A∩C)=P(A)P(C).

However, I have no idea how to solve this. I attempted to apply the probability rules I know but got nowhere.